LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT2MC01 – ALGEBRA

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1.	**Answer the following**
a)	Let G be a group and $a \in G$. Define the normalizer of a.
b)	Write the center of a cyclic group.
c)	Give an example for an infinite integral domain.
d)	Define simple extension.
e)	Write a field with order 27.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2.	**MCQ**
a)	Let order of G is 121. Then G is a) Non abelian b) abelian c) cyclic d) Simple
b)	Which of the following is cyclic a) $Z_{10} \times Z_{10}$ b) $Z_2 \times Z_{10}$ c) $Z_4 \times Z_{10}$ d) $Z_5 \times Z_7$
c)	The polynomial x^2+1 is reducible over a) Z b) Q c) Z_2 d) R
d)	Degree of $Q(\sqrt{7})$ over Q is a) 1 b) 2 c) 3 d) 4
e)	Any finite field of order 11 is isomorphic to . a) Q b) R c) Z_{121} d) Z_{11}
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3.	If $O(G) = p^n$ where p is a prime number, then show that $Z(G) \neq (e)$.
4.	Show that if G is a finite group, then $c_a = \frac{o(G)}{o(N(a))}$ where $a \in G$.
5.	Prove that the product of two primitive polynomial is primitive.
6.	Show that a polynomial of degree n over a field can have at most n roots in any extension field.
7.	Show that for any prime number p and every positive integer m there exists a field having p^m elements.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	State and prove Cauchy's theorem.
9.	Show that any group of order $11^2 13^2$ is abelian.
10.	Discuss about $\frac{Q[X]}{\langle x^3-2\rangle}$. Also write the multiplicative inverse of $x+\langle x^3-2\rangle$.
11.	Show that the multiplicative group of nonzero elements of a finite field is cyclic.

SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 15 = 15)**
12.	Show that any finite abelian group is internal direct product of its Sylow subgroups. Apply it for an abelian group with order 30.
13.	Discuss about Eisenstein Criterion. Also verify the irreducibility of $x^5 + 10x^4 - 20x^3 + 5x^2 - 10$ over Q and Z.
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14.	a) Write about first and second part of Sylow's theorems. b) Let G denote the group S_4xS_3. Then prove that 2-sylow subgroup of G and 3-sylow subgroup of G are not normal. (14 + 6)
15.	a) Let K be a normal extension of F and H be a subgroup of $G(K, H)$. Prove that $[K: K_H] = o(H)$ and $H = G(K: K_H)$. b) Construct a field extension with order 6 over Q. Also write its basis over Q. (14 + 6)

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